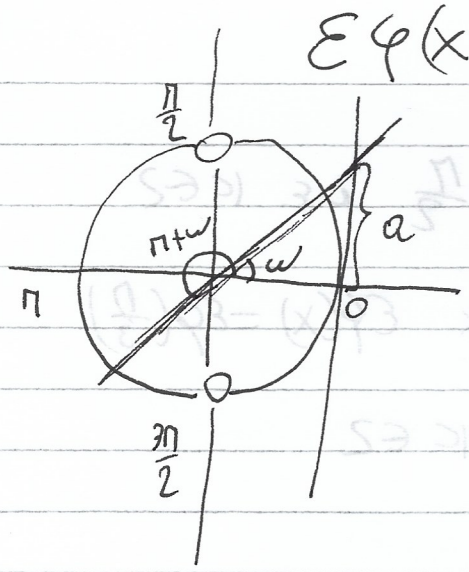
# $\varepsilon\varphi(x) = a$




$\varepsilon\varphi(x) = \varepsilon\varphi\omega$ άρα $x = K\pi + \omega$, $K \in \mathbb{Z}$

1) πρέπει $x \neq 2K\pi + \pi/2$, $K \in \mathbb{Z}$

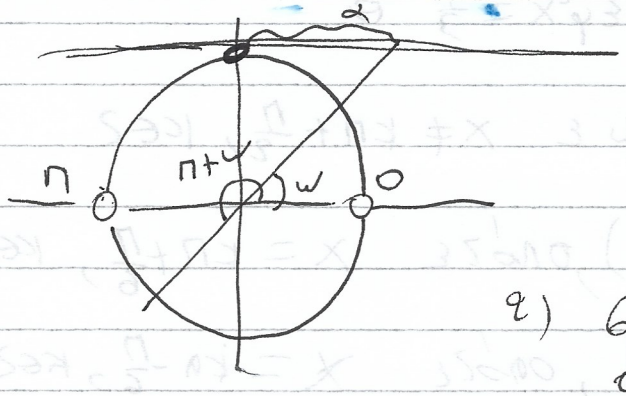
2) $\left.\begin{matrix} \varepsilon\varphi(x) = a \\ a = \varepsilon\varphi(\omega) \end{matrix}\right\}$ $\varepsilon\varphi(x) = \varepsilon\varphi(\omega)$

$x = K\pi + \omega$, $K \in \mathbb{Z}$

Χρησιμοποιώντας σχέσεις

| | |
|---|---|
| $0 = \varepsilon\varphi(0)$ | $\sqrt{3} = \varepsilon\varphi(\frac{\pi}{3})$ |
| $\sqrt{3}/3 = \varepsilon\varphi(\frac{\pi}{6})$ | $-\varepsilon\varphi\omega = \varepsilon\varphi(-\omega)$ |
| $1 = \varepsilon\varphi(\frac{\pi}{4})$ | $\sigma\varphi(\omega) = \varepsilon\varphi(\frac{\pi}{2} - \omega)$ |

---




$\sigma\varphi(x) = a$

1) $x \neq K\pi$, $K \in \mathbb{Z}$

2) $\left.\begin{matrix} \sigma\varphi x = a \\ a = \sigma\varphi\omega \end{matrix}\right\}$ $\sigma\varphi x = \sigma\varphi\omega$

$x = K\pi + \omega$, $K \in \mathbb{Z}$

Χρησιμοποιώ τις σχέσεις

| | |
|---|---|
| $\sqrt{3} = \sigma\varphi(\frac{\pi}{6})$ | $0 = \sigma\varphi(\frac{\pi}{2})$ |
| $1 = \sigma\varphi(\frac{\pi}{4})$ | $-\sigma\varphi\omega = \sigma\varphi(-\omega)$ |
| $\sqrt{3}/3 = \sigma\varphi(\frac{\pi}{3})$ | $\varepsilon\varphi\omega = \sigma\varphi(\frac{\pi}{2} - \omega)$ |

# Λυμένες ασκήσεις

→ $\varepsilon\varphi(x) = \sqrt{3}$ 1) πρέπει $x \neq k\pi + \frac{\pi}{2}$ με $k \in \mathbb{Z}$

2) $\left.\begin{array}{l} \varepsilon\varphi(x) = \sqrt{3} \\ \sqrt{3} = \varepsilon\varphi\left(\frac{\pi}{3}\right) \end{array}\right\}$ άρα $\varepsilon\varphi(x) = \varepsilon\varphi\left(\frac{\pi}{3}\right)$

οπότε $x = k\pi + \frac{\pi}{3}$ με $k \in \mathbb{Z}$

→ $\sigma\varphi(2x) = 1$ 1) πρέπει $2x \neq k\pi \Leftrightarrow x \neq \frac{k\pi}{2}$, με $k \in \mathbb{Z}$

2) $\left.\begin{array}{l} \sigma\varphi(2x) = 1 \\ \text{όμως } 1 = \sigma\varphi\left(\frac{\pi}{4}\right) \end{array}\right\}$ $\sigma\varphi(2x) = \sigma\varphi\left(\frac{\pi}{4}\right)$

Άρα $2x = k\pi + \frac{\pi}{4} \Leftrightarrow x = \frac{k\pi}{2} + \frac{\pi}{8}$, με $k \in \mathbb{Z}$

→ $3\varepsilon\varphi^2 x - 1 = 0 \Leftrightarrow 3\varepsilon\varphi^2 x = 1 \Leftrightarrow \varepsilon\varphi^2 x = \frac{1}{3} \Leftrightarrow$

$\varepsilon\varphi^2(x) = \frac{3}{9} \Leftrightarrow \varepsilon\varphi x = \pm\frac{\sqrt{3}}{3}$ με $x \neq k\pi + \frac{\pi}{2}$, $k \in \mathbb{Z}$

1ον $\varepsilon\varphi(x) = \frac{\sqrt{3}}{3} \Leftrightarrow \varepsilon\varphi x = \varepsilon\varphi\left(\frac{\pi}{6}\right)$, οπότε $x = k\pi + \frac{\pi}{6}$, $k \in \mathbb{Z}$

2ον $\varepsilon\varphi(x) = -\frac{\sqrt{3}}{3} \Leftrightarrow \varepsilon\varphi x = \varepsilon\varphi\left(-\frac{\pi}{6}\right)$, οπότε $x = k\pi - \frac{\pi}{6}$, $k \in \mathbb{Z}$