

① Απόσταση σημείου από ευθεία

$$d(A, E) = \frac{|\alpha x_0 + \beta y_0 + \gamma|}{\sqrt{\alpha^2 + \beta^2}}$$

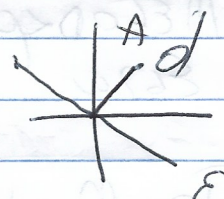
$$E: \alpha x + \beta y + \gamma = 0$$

$$A(x_0, y_0)$$

Παράδειγμα 1

1^ο $A(2, 2)$

$$E: x + y = 0$$

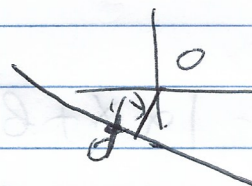


$$d(A, E) = \frac{|2 + 2|}{\sqrt{1 + 1}} = \frac{4}{\sqrt{2}} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$$

2^ο

$$A(0, 0)$$

$$E: 3x + 4y + 7 = 0$$



$$d(A, E) = \frac{|3 \cdot 0 + 4 \cdot 0 + 7|}{\sqrt{3^2 + 4^2}} = \frac{7}{5}$$

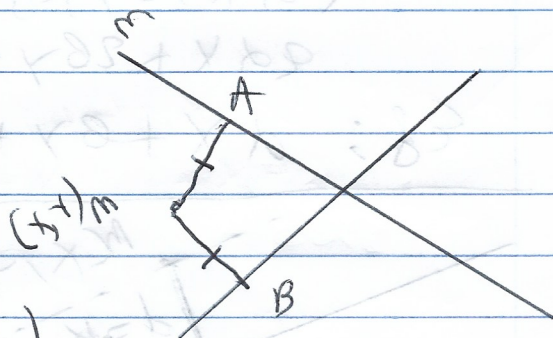
② Διχοτομία - χωρίο

$$E_1: \alpha_1 x + \beta_1 y + \gamma_1 = 0$$

$$E_2: \alpha_2 x + \beta_2 y + \gamma_2 = 0$$

$$A(x_0, y_0)$$

$$m_A = m_B \Leftrightarrow d(m, E_1) = d(m, E_2)$$



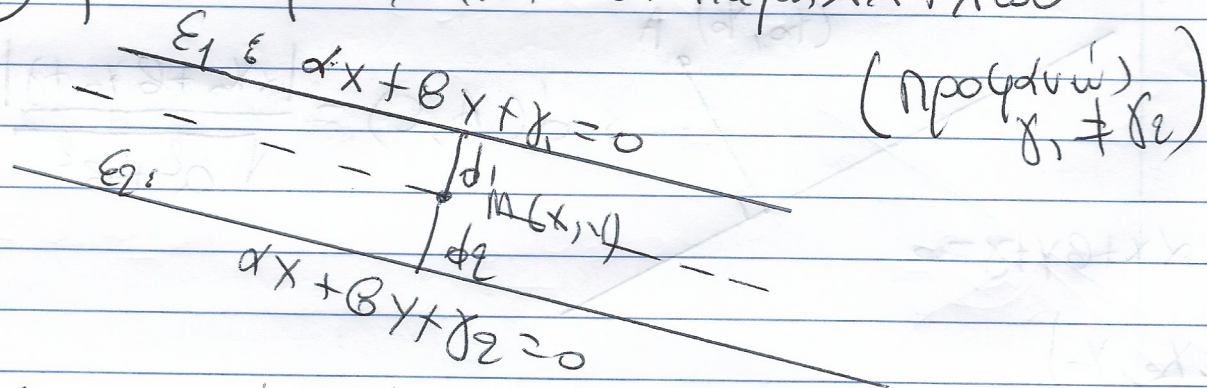
$$\Leftrightarrow \frac{|\alpha_1 x_0 + \beta_1 y_0 + \gamma_1|}{\sqrt{\alpha_1^2 + \beta_1^2}} = \frac{|\alpha_2 x_0 + \beta_2 y_0 + \gamma_2|}{\sqrt{\alpha_2^2 + \beta_2^2}} \Leftrightarrow$$

$$\sqrt{\alpha_2^2 + \beta_2^2} |\alpha_1 x_0 + \beta_1 y_0 + \gamma_1| = \sqrt{\alpha_1^2 + \beta_1^2} |\alpha_2 x_0 + \beta_2 y_0 + \gamma_2|$$

$$\Leftrightarrow \sqrt{\alpha_2^2 + \beta_2^2} (\alpha_1 x_0 + \beta_1 y_0 + \gamma_1) = + \sqrt{\alpha_1^2 + \beta_1^2} (\alpha_2 x_0 + \beta_2 y_0 + \gamma_2)$$

$$\Leftrightarrow \sqrt{\alpha_2^2 + \beta_2^2} (\alpha_1 x_0 + \beta_1 y_0 + \gamma_1) = - \sqrt{\alpha_1^2 + \beta_1^2} (\alpha_2 x_0 + \beta_2 y_0 + \gamma_2)$$

(3) μεσοπαράλληλη 2 παραλλήλων



Μεσοπαράλληλη:
Εστὼ $m(x, y)$

Τὸ m ἡ ἀπόστασις ἀπὸ τῶν $\varepsilon_1, \varepsilon_2 \Leftrightarrow$

$$d(m, \varepsilon_1) = d(m, \varepsilon_2) \Leftrightarrow \frac{|ax + by + \gamma_1|}{\sqrt{a^2 + b^2}} = \frac{|ax + by + \gamma_2|}{\sqrt{a^2 + b^2}}$$

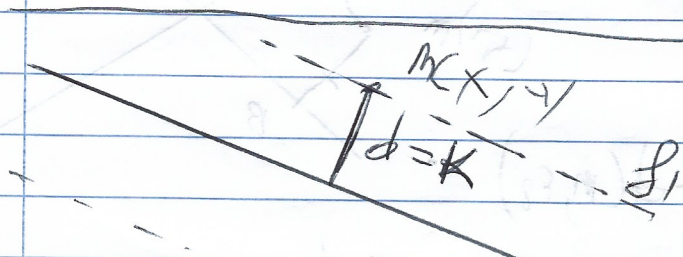
$$\Leftrightarrow |ax + by + \gamma_1| = |ax + by + \gamma_2|$$

$$\left\{ \begin{array}{l} ax + by + \gamma_1 = ax + by + \gamma_2 \quad \text{ἀποκρίνεται} \\ ax + by + \gamma_1 = -ax - by - \gamma_2 \end{array} \right.$$

$$ax + by + \gamma_1 = -ax - by - \gamma_2 \Leftrightarrow$$

$$2ax + 2by + (\gamma_1 + \gamma_2) = 0 \Leftrightarrow$$

$$\varepsilon_f: ax + by + \frac{\gamma_1 + \gamma_2}{2} = 0$$



Πᾶς βρεῖτε τὸν Γ.Τ

τὴν m , ὅπου ἡ ἀπόστασις ἀπὸ τῶν $\varepsilon_1, \varepsilon_2$

$d = K$ ἀπὸ τὴν

$$\varepsilon: ax + by + \gamma = 0$$

$$d(m, \varepsilon) = K \Leftrightarrow \frac{|ax + by + \gamma|}{\sqrt{a^2 + b^2}} = K \Leftrightarrow |ax + by + \gamma| = K \sqrt{a^2 + b^2}$$

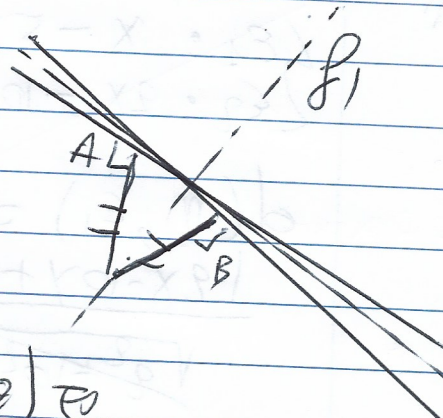
$$\left\{ \begin{array}{l} \varepsilon_1: ax + by + \gamma = K \sqrt{a^2 + b^2} \\ \varepsilon_2: ax + by + \gamma = -K \sqrt{a^2 + b^2} \end{array} \right.$$

$$\varepsilon_2: ax + by + \gamma = -K \sqrt{a^2 + b^2}$$

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Ταράδεξις: $\epsilon_1: 3x + 4y - 5 = 0$
 $\epsilon_2: 5x + 12y - 17 = 0$

Να βρείτε το διχοτόμο.



Εστω M τοχάιο γηρείο
 τη διχοτόμου. Τότε

$$MA = MB \Leftrightarrow d(M, \epsilon_1) = d(M, \epsilon_2) \Leftrightarrow$$

$$\frac{|3x + 4y - 5|}{\sqrt{3^2 + 4^2}} = \frac{|5x + 12y - 17|}{\sqrt{5^2 + 12^2}} \Leftrightarrow$$

$$\frac{|3x + 4y - 5|}{5} = \frac{|5x + 12y - 17|}{13} \Leftrightarrow$$

$$13|3x + 4y - 5| = 5|5x + 12y - 17| \Leftrightarrow$$

$$|39x + 52y - 65| = |25x + 60y - 85|$$

$$\begin{aligned} f_1: 39x + 52y - 65 &= 25x + 60y - 85 \Leftrightarrow \\ 14x - 8y + 20 &= 0 \end{aligned}$$

$$\begin{aligned} f_2: 39x + 52y - 65 &= -25x - 60y + 85 \Leftrightarrow \\ 64x + 112y - 150 &= 0 \end{aligned}$$

Ασκήσιν 6 ευλ : 1) Να βρείτε το διχοτόμο
 των ~~επί~~ ευθύνων $\epsilon_1: x + 2y - 2 = 0$
 $\epsilon_2: 6x + 3y - 9 = 0$

2) Να βρείτε το διχοτόμο των
 $\epsilon_1: x + y = 0$
 $\epsilon_2: x - y = 0$

ΜΕ 30 ΜΑ 98 ΜΗ 114

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Να βρείτε την μεσοκράδα της των

$$\begin{cases} \varepsilon_1: x - 5y + 6 = 0 \\ \varepsilon_2: 2x - 10y + 6 = 0 \end{cases} \Rightarrow \begin{cases} 2x - 10y + 12 = 0 \\ 2x - 10y + 6 = 0 \end{cases}$$

Εδώ $m(x, y)$
 $d(m, \varepsilon_1) = d(m, \varepsilon_2) \Leftrightarrow$

$$\frac{|2x - 10y + 12|}{\sqrt{2^2 + 10^2}} = \frac{|2x - 10y + 6|}{\sqrt{2^2 + 10^2}} \quad (\Rightarrow)$$

$$|2x - 10y + 12| = |2x - 10y + 6|$$

$$\swarrow \quad \cancel{2x - 10y + 12} = \cancel{2x - 10y} + 6, \text{ < σφάλμα >}$$

$$\swarrow \quad 2x - 10y + 12 = -2x + 10y - 6 \Leftrightarrow 4x - 20y + 18 = 0 \\ \Leftrightarrow 2x - 10y + 9 = 0$$

Αδυναμία Να βρείτε την μεσοκράδα της των

$$\varepsilon_1: 2x + 6y - 8 = 0, \quad \varepsilon_2: 4x + 12y + 10 = 0$$

Ευθεία
 μεσοκράδα

Δίνεται η $4x - 3y + 5 = 0$ (c)

Να βρείτε τον Γ.Τ. των ευθειών $m(x, y)$
 των οποίων, $0 < x < 0.901$, $0 < y < 0.901$

$$d(m, \varepsilon) = 2$$

$$d(m, \varepsilon) = 2 \Leftrightarrow$$

$$\frac{|4x - 3y + 5|}{\sqrt{4^2 + 3^2}} = 2 \Leftrightarrow |4x - 3y + 5| = 10$$

$$\swarrow \quad g_1: 4x - 3y + 5 = 10 \Leftrightarrow 4x - 3y - 5 = 0$$

$$\swarrow \quad g_2: 4x - 3y + 5 = -10 \Leftrightarrow 4x - 3y - 15 = 0$$

Αδυναμία: $m(x, y)$ $\varepsilon: 5x + 12y + 17 = 0, d(m, \varepsilon) = 3$

Γ.Τ. m: