

Εξίσωση 6ης που ανήχονταν σε 2^{ου} βαθμού

$$\begin{aligned} \underline{1^{\text{η}} \text{ μορφή}} & \quad \alpha x^{2v} + \beta x^v + \gamma = 0 \\ \rightarrow \text{Θέζω} & \quad x^v = y, \text{ οπότε } x^{2v} = y^2 \end{aligned} \quad \left. \vphantom{\begin{aligned} \underline{1^{\text{η}} \text{ μορφή}} & \quad \alpha x^{2v} + \beta x^v + \gamma = 0 \\ \rightarrow \text{Θέζω} & \quad x^v = y, \text{ οπότε } x^{2v} = y^2 \end{aligned}} \right\}$$

Η αρχική εξίσωση γράφεται:

$$\alpha y^2 + \beta y + \gamma = 0$$

$$\Delta =$$

$$\rho_1 =$$

$$\rho_2 =$$

$$\rightarrow y = \rho_1 \Rightarrow x^v = \rho_1 \dots \dots$$

$$\rightarrow y = \rho_2 \Rightarrow x^v = \rho_2 \dots \dots$$

$$\underline{2^{\text{η}} \text{ μορφή}} \quad \alpha (x-k)^2 + \beta |x-k| + \gamma = 0$$

$$\rightarrow \text{Θέζω} \quad |x-k| = y \text{ οπότε } (x-k)^2 = y^2$$

Η αρχική εξίσωση γράφεται

$$\alpha y^2 + \beta y + \gamma = 0, \quad \Delta =$$

$$\rho_1 =$$

$$\rho_2 =$$

$$\rightarrow y = \rho_1 \Rightarrow |x-k| = \rho_1 \dots \dots$$

$$\rightarrow y = \rho_2 \Rightarrow |x-k| = \rho_2 \dots \dots$$

Παράδειγμα 2

$$1/x^4 - 8x^2 - 9 = 0 \quad (I)$$

Παρατηρώ ότι $(x^2)^2 = x^4$

Αν θέσω $x^2 = y$, τότε $x^4 = y^2$

$$\text{Η (I) γράφεται } y^2 - 8y - 9 = 0$$

$$\Delta = 64 + 36 = 100 > 0$$

$$y_1 = \frac{8+10}{2} = 9$$

$$\rightarrow y = 9 \Leftrightarrow x^2 = 9 \Leftrightarrow x = \pm 3$$

$$y_2 = \frac{8-10}{2} = -1$$

$$\rightarrow y = -1 \Leftrightarrow x^2 = -1 \text{ αδύνατο}$$

$$2) (x-5)^2 - 4|x-5| - 5 = 0 \quad (II)$$

Παρατηρώ ότι $|x-5|^2 = (x-5)^2$

Αν θέσω $|x-5| = y$, τότε $|x-5|^2 = y^2$

$$\text{Η (II) γράφεται } y^2 - 4y - 5 = 0$$

$$\Delta = 16 + 20 = 36$$

$$y_1 = \frac{4-6}{2} = -1$$

$$y_2 = \frac{4+6}{2} = 5$$

$$1^{\text{η}} y = -1 \Leftrightarrow |x-5| = -1 \text{ αδύνατο}$$

$$2^{\text{η}} y = 5 \Leftrightarrow |x-5| = 5$$

$$x-5 = 5 \Leftrightarrow x = 10$$

$$x-5 = -5 \Leftrightarrow x = 0$$

$$3) (x + \frac{1}{x})^2 - 3(x + \frac{1}{x}) + 2 = 0 \quad \text{Θέτω } x + \frac{1}{x} = y \quad \text{με } x \neq 0$$

$$y^2 - 3y + 2 = 0 \quad \Delta = 9 - 8 = 1 > 0$$

$$y_1 = \frac{3+1}{2} = 2$$

$$y_2 = \frac{3-1}{2} = 1$$

$$1^{\text{η}} y = 2 \Leftrightarrow x + \frac{1}{x} = 2$$

$$\Rightarrow x^2 + 1 = 2x \Leftrightarrow x^2 - 2x + 1 = 0$$

$$(x-1)^2 = 0 \Leftrightarrow \underline{x=1}$$

$$2^{\text{η}} y = 1 \Leftrightarrow x + \frac{1}{x} = 1 \Leftrightarrow x^2 + 1 = x \Leftrightarrow x^2 - x + 1 = 0$$

$$\Delta < 0 \text{ αδύνατο}$$